American University of Beirut

STAT 230

Introduction to Probability and Random Variables

Spring 2008

quiz # 1

Name: ID #:

circle your section please: **1 (TTH 8 am)** **2 (TTH 2 pm)**

Exercise 1 *(20 points)*

a. 3 history books, 4 math books, and 2 biology books are to be arranged in a row. Find the probability that books of the same topic are side by side.

b. In a string of 12 Christmas tree light bulbs, 3 are defective. The bulbs are selected at random and tested, one at a time, until the third defective bulb is found. Find the probability that the third defective bulb is the tenth bulb tested.

c. A drawer contains four black, six white, and eight olive socks. Two socks are selected at random from the drawer. Find the probability that both socks are olive if it is known that they are of the same color.

Exercise 2 *(10 points)* Let A and B be two independent events with $P(A) = P(B) = 1/2$. Find $P((A \cap B') \cup (A' \cap B))$.

Exercise 3 *(15 points)* Player A has entered a golf tournament but it is not certain whether player B will enter. Player A has probability 1/6 of winning the tournament if players B enters and probability 3/4 of winning if player B does not enter the tournament. The probability that player B enters is 1/3. If player A wins the tournament, find the probability that player B entered the tournament.

Exercise 4 *(20 points)* A, B, and C take turns rolling a die, A first, then B, then C, then A, and so on. The first one to roll a 5 or 6 wins. Find the probability that A will win.

Exercise 5 *(15 points)* A secretary typed 10 letters and addressed 10 envelopes. For some reason the letters were put into envelopes at random. Find the probability of at least one letter being put into the correct envelope.

Exercise 6 *(20 points)* Let X be a random variable with pdf

$$P(X = k) = \frac{1}{(k+1)(k+2)}, k = 0, 1, 2, \ldots$$

Find the conditional probability of $X \geq 4$, given that $X \geq 1$.
Show that $E(X)$ does not exist.

good luck

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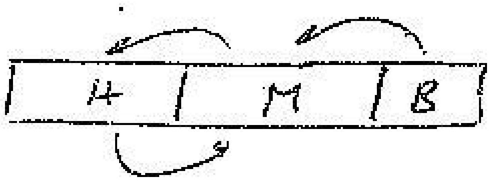
1) a) $3! \times \frac{4! \times 3! \times 2!}{9!}$



b) $\frac{C_3^2 \times C_9^7}{C_{12}^9} \times \frac{1}{3}$.

c) $A = \{\text{both are olive}\} = \{OO\}$

$B = \{\text{both are of the same color}\} = \{OO, BB, WW\}$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{C_8^2 / C_{18}^2}{C_8^2/C_{18}^2 + C_6^2/C_{18}^2 + C_4^2/C_{18}^2} = \frac{C_8^2}{C_8^2 + C_6^2 + C_4^2}$$

2) $$P((A\cap B') \cup (A'\cap B)) = P(A\cap B') + P(A'\cap B) + P(\underbrace{A\cap B' \cap A' \cap B}_{\phi})$$

$$\overset{ind.}{=} P(A).P(B') + P(A').P(B) + 0$$

$$= \frac{1}{2} * \frac{1}{2} + \frac{1}{2} * \frac{1}{2} = \frac{1}{2}.$$

3) $A = \{A \text{ wins the tournament}\}$

$B = \{B \text{ enters}\}$

$$P(B|A) \underset{Bayes}{=} \frac{P(A|B)\,P(B)}{P(A|B)P(B) + P(A|\bar{B})\,P(\bar{B})} = \frac{\frac{1}{6}\times\frac{1}{3}}{\frac{1}{6}\times\frac{1}{3} + \frac{3}{4}\times\frac{2}{3}} = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{6}{4}} = \frac{4}{4+36} = \frac{1}{10}.$$

4) $$P(A\text{ wins}) = \sum_{n=0}^{+\infty} P(\underbrace{\bar{A}\bar{B}\bar{C}\bar{A}\bar{B}\bar{C} \ldots \bar{A}\bar{B}\bar{C}}_{(3n)\text{ times}} A) \underset{ind.}{=} \sum_{n=0}^{+\infty} \left(\frac{4}{6}\right)^{3n} \times \frac{2}{6}$$

$$= \frac{2}{6} \times \sum_{n=0}^{+\infty} \left(\frac{8}{27}\right)^n = \frac{1}{3} \times \frac{1}{1-\frac{8}{27}} = \frac{9}{19}.$$

5) $A = \{$ at least one letter is in the correct envelope $\}$

$$P(A) = 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!} - \frac{1}{6!} + \frac{1}{7!} - \frac{1}{8!} + \frac{1}{9!} - \frac{1}{10!}.$$

- matching formula

6) $$P(X \geq 4 \mid X \geq 1) = \frac{P(X \geq 4 \cap X \geq 1)}{P(X \geq 1)} = \frac{P(X \geq 4)}{P(X \geq 1)} = \frac{\sum_{k=4}^{+\infty} \frac{1}{(k+1)(k+2)}}{\sum_{k=1}^{+\infty} \frac{1}{(k+1)(k+2)}}$$

$$= \frac{\frac{1}{5}}{\frac{1}{2}} = \frac{2}{5}$$

telescoping series

$$\left(\frac{1}{(k+1)(k+2)} = \frac{1}{k+1} - \frac{1}{k+2}\right)$$

$$E(X) = \sum_{k=0}^{+\infty} k \cdot P(X=k) = \sum_{k=0}^{+\infty} \frac{k}{(k+1)(k+2)}$$ divergente series

by LCT with $\sum_{k=1}^{+\infty} \frac{1}{k}$